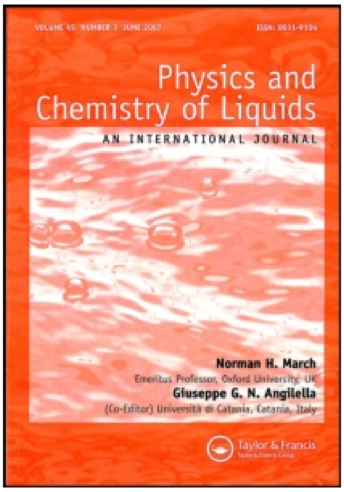

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ON THE STAGNATION POINT FLOW OF A SPECIAL CLASS OF NON-NEWTONIAN FLUIDS

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The two-dimensional boundary layer equations for a class of non-Newtonian fluids, for which the apparent viscosity can be expressed as a polynomial in the second scalar invariant of the rate of strain tensor, have been derived. These equations have been employed to analyse the flow near a stagnation point over a stationary impermeable wall. The non-Newtonian effects on the boundary layer velocity profile and the wall skin friction have been studied, and compared with the corresponding Newtonian fluid. The fluid velocity in the boundary layer has been shown to be retarded by the non-Newtonian effect while the skin friction increases proportionate to it.

Keywords: Non-Newtonian fluid; two-dimensional flow; stagnation point; boundary layer; skin friction

1. INTRODUCTION

As is known, the extent to which the non-Newtonian properties of the fluids influence the flow features in various applications in chemical, biochemical and mineral processing industries varies from one application to another. To a large measure, this depends on the way the apparent viscosity of the fluid varies with the shear rate. For

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instance, fluids are encountered in applications for which the apparent viscosity may decrease or increase with increasing shear rate. Our interest in this note is to investigate the non-Newtonian effects on the stagnation point flow when the viscosity increases with shear rate which may occur, for example, in processing of highly concentrated suspensions or pastes [1 – 3]. A possible constitutive equation of such a fluid can be represented as [4]

$$\tau_{ij} = \mu(I_1, I_2, I_3)e_{ij} \tag{1}$$

where I_1, I_2, and I_3 are the scalar invariants of the rate of strain tensor. For two-dimensional flows, I_1 and I_3 are identically equal to zero; therefore, $\mu = \mu(I_2)$. We further assume [5] that $\mu(I_2)$ can be expressed as a polynomial in I_2:

$$\mu(I_2) = \mu_0 + \mu_1 I_2 + \mu_2 I_2^2 + \cdots, \quad (\mu_i > 0) \tag{2}$$

which exhibits a kind of shear thickening dilatant behaviour. In this work, we restrict ourselves up to the first order term in I_2 in the expansion of μ. Thus, we write

$$\mu = \mu_0 + \mu_1 I_2 \tag{3}$$

where μ_0 is the conventional Newtonian viscosity coefficient and μ_1 (> 0) is a parameter characterising the non-Newtonian behaviour of the fluid.

It is worth mentioning here that although flow analyses of non-Newtonian fluids using different constitutive equations have been extensively reported in literature, little attention has been paid to the dynamics of fluid models described by the constitutive Eq. (2). As the visco-plastic models have been found not to sufficiently explain the behaviour of certain fluids such as concentrated suspensions, the analyses of models described by Eq. (2) are of interest. Thus the present work is undertaken in order to study the boundary layer flow of the non-Newtonian fluid model given by Eq. (3) near a two-dimensional stagnation point over a stationary wall. The governing equations have been reduced to a non-linear differential equation of third order using suitable transformations. The resulting two-point boundary value problem has been solved numerically using a shooting method.

2. GOVERNING EQUATIONS AND THEIR SOLUTION

We consider the steady, laminar flow of the non-Newtonian fluid with velocity components $u = u(x, y)$ and $v = v(x, y)$ near a two-dimensional stagnation point over a solid boundary at $y = 0$. Transforming the tensor components τ_{ij} into physical components in cartesian coordinates and using the two-dimensional equations of conservation of mass and momentum, the boundary layer equations based on the usual order of magnitude approach can be derived in the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2} + 3\Lambda\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} \tag{5}$$

$$\frac{\partial p}{\partial y} = 0 \tag{6}$$

where p is the pressure, U the mainstream velocity, $\nu = \mu_0/\varrho$, $\Lambda = \mu_1/\varrho$ and ϱ is the density. The boundary conditions for the velocity field are

$$\begin{aligned} &u = 0, \quad v = 0 \quad \text{at } y = 0 \\ &u \to U \quad \text{as } y \to \infty. \end{aligned} \tag{7}$$

In order to solve the boundary layer Eqs. (4)–(6) subject to the conditions (7), we let

$$\begin{aligned} &U = U_1 x, \quad \text{(where } U_1 \text{ is a constant)}, \\ &\eta = (U_1/\nu)^{1/2} y \\ &\psi = (\nu U_1)^{1/2} x f(\eta) \\ &u = \partial\psi/\partial y, \quad v = -\partial\psi/\partial x. \end{aligned} \tag{8}$$

It can be verified that the continuity equation is automatically satisfied by ψ. The velocity components u and v now become

$$u = Uf'(\eta), \quad v = -(\nu U_1)^{1/2} f(\eta). \tag{9}$$

Using the expressions in Eqs. (8) and (9) in Eq. (5), we obtain the differential equation

$$f''' + ff'' - (f')^2 + 1 + \beta x^{*^2}(f'')^2 f''' = 0 \tag{10}$$

where $x^* = x/L$, L a characteristic length scale and $\beta = (3\Lambda U_1^3 L^2)/\nu^2$ is the parameter characterising the ratio of the non-Newtonian effect to the Newtonian viscous effect. In Eq. (10), the primes denote differentiation with respect to η. The transformed boundary conditions are

$$f(0) = 0,\ f'(0) = 0,\ f'(\infty) = 1. \tag{11}$$

In the following, we shall assume that x^* is a parameter, rather than an independent variable. In other words, we shall seek local similarity solution of Eq. (10) corresponding to small values of the parameter $c\,(= x^{*^2})$ near the stagnation point. Equations (10) and (11) thus describe a well-posed boundary value problem. This may be contrasted with similar studies in visco-elastic fluids [7–9] in which the corresponding velocity functions were shown to be governed by equations whose orders did not match the number of physical boundary conditions. However, they overcame this difficulty by resorting to a perturbation technique and reducing the governing non-linear equations into systems of equations in each of which the order of equation matched the number of boundary conditions. When $\beta = 0$, Eq. (10) reduces to the well-known equation for viscous fluids.

Equation (10) with the boundary conditions (11) can be solved using shooting method provided an estimate for $f''(0)$ is available. The accuracy and the convergence rate of the resulting solution will depend on the accuracy of this estimate. In order to generate an acceptable initial guess for $f''(0)$, we first consider a perturbation solution of Eq. (10) with respect to the parameter β, assumed small. To this end, we write

$$f(\eta) = f_0(\eta) + \beta f_1(\eta) + \beta^2 f_2(\eta) + \beta^3 f_3(\eta) + \cdots. \tag{12}$$

Using Eq. (12) in Eq. (10), and collecting terms of equal orders in β, we get a set of ordinary differential equations governing f_n, $(n = 0, 1, 2, \ldots)$. The order of terms to be considered in the perturbed quantities depends on the accuracy desired. Sarpkaya and Rainey [7] have shown that more accurate results can be obtained by taking higher order terms. In this work we have taken terms up to and including the second order. The function $f_0(\eta)$ is governed by the non-linear equation

$$f_0''' + f_0 f_0'' - (f_0')^2 + 1 = 0 \tag{13}$$

subject to

$$f_0(0) = 0,\ f_0'(0) = 0,\ f_0'(\infty) = 1. \tag{13a}$$

The equations governing the higher order functions can be written in the matrix form

$$\mathbf{a}\,\mathbf{X} = \mathbf{b} \tag{14}$$

where

$$\begin{aligned} \mathbf{a} &= (1\ f_0\ \ -2f_0'\ \ f_0''), \quad \mathbf{X} = (\mathbf{F}'''\ \mathbf{F}''\ \mathbf{F}'\ \mathbf{F})^{\mathrm{T}}, \\ \mathbf{F} &= (f_1\ \ f_2\ f_3\ \ldots), \quad \mathbf{b} = (K_1\ K_2\ K_3\ \ldots) \end{aligned}$$

and K_1, K_2, ... are known functions. The boundary conditions for Eq. (14) are

$$f_n(0) = f_n'(0) = f_n'(\infty) = 0, \quad (n \geq 1). \tag{14a}$$

As noted before, we have considered the effects of terms up to β^2. Besides Eq. (13), other equations and their boundary conditions in this case can be expressed in the explicit forms

$$f_1''' + f_0 f_1'' - 2f_0' f_1' + f_0'' f_1 = -c(f_0'')^2 f_0''' \tag{15}$$

$$f_1(0) = f_1'(0) = f_1'(\infty) = 0 \tag{15a}$$

$$f_2''' + f_0 f_2'' - 2f_0' f_2' + f_0'' f_2 = (f_1')^2 - f_1 f_1' - c\{(f_0'')^2 f_1''' + 2f_0'' f_0''' f_1''\} \tag{16}$$

$$f_2(0) = f_2'(0) = f_2'(\infty) = 0. \tag{16a}$$

In many applications, the prediction of the effect of the non-Newtonian parameter on the local wall shear stress is of importance. For the model considered here, the non-dimensional skin friction coefficient τ is given by

$$\tau = \frac{(\tau_{xy})_{y=0}}{\varrho L \nu^{1/2} U_1^{3/2}} = c^{1/2}\left[f''(0) + \frac{1}{3}\beta c\{f''(0)\}^3\right]. \tag{17}$$

When $\beta = 0$, Eq. (17) yields the skin friction for the corresponding Newtonian case [6].

In order to obtain the velocity profiles, Eqs. (13), (15) and (16) were first integrated numerically using a shooting method. The approximate solutions so obtained were used to solve the original Eq. (10). The improved value of $f''(0)$ obtained from the perturbation method was seen to accelerate the convergence of the solution to Eq. (10). The double precision arithmetic was used in the computations. For graphing purposes, the computed results were seen to be insensitive to moderate integration step sizes; however, for tabulated values, a step length of $\Delta\eta = 0.001$ was used. The results obtained correspond to the value of c equal to 0.5.

3. RESULTS

The velocity distribution in the boundary layer is shown in Figure 1. The curve for β equals to zero corresponds to the Newtonian profile. The velocity tends to decrease as β is increased. This shows that the non-Newtonian effect of the type considered herein retards the fluid motion. The non-Newtonian effect is more pronounced at some distance from the wall before velocity profiles eventually merge smoothly with the corresponding Newtonian flow. For small values of β, the boundary layer thickness has almost a constant profile. Moreover, the results here do not show the overshooting phenomenon observed in visco-elastic fluids [8, 9].

The values of the coefficient of skin friction τ at the boundary have been tabulated for a range of values of the non-Newtonian parameter β (see Tab. I). In this table, τ_e denotes the skin friction coefficient calculated from the solution of Eq. (10) while τ_a denotes the

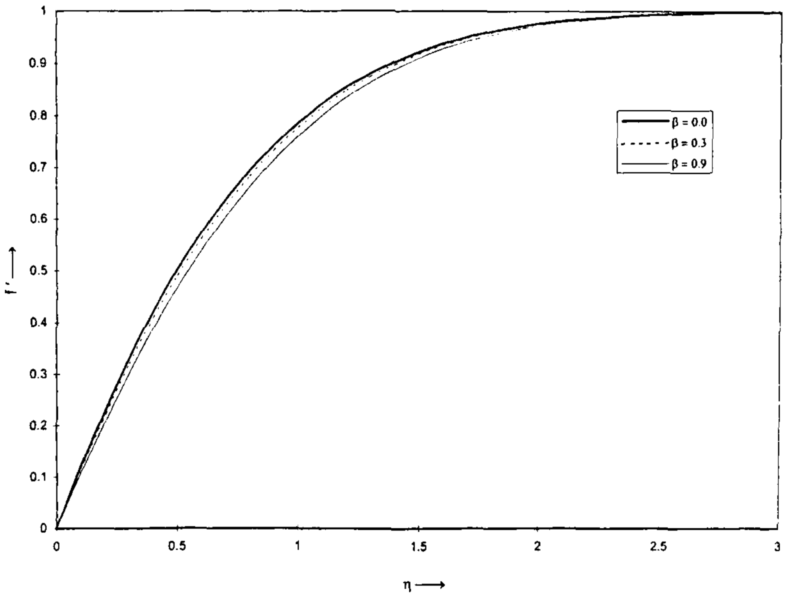

FIGURE 1 Velocity distribution in the boundary layer.

TABLE I Skin friction τ

β	τ_e	τ_a
0.0	0.8716	0.8716
0.1	0.8771	0.8772
0.2	0.8824	0.8831
0.3	0.8875	0.8899
0.4	0.8923	0.8979
0.5	0.8969	0.9077
0.6	0.9014	0.9196
0.7	0.9057	0.9341
0.8	0.9099	0.9516
0.9	0.9140	0.9726

corresponding values obtained using the perturbed solution. The stress at the wall shows increasing trend with increase in the parameter β. As should be expected, the difference between the values of τ_a and τ_e is negligible for small β; however, as β is increased, the error in τ_a increases in comparison with τ_e. Regarding the perturbation solution, it is worth mentioning here that the sum of zeroth and first order terms in the perturbation analysis did not yield stress behaviour at the boundary comparable with the exact numerical solution. However, on taking the next higher order term, τ_a showed increasing pattern with increase in the non-Newtonian effect. The values of τ_a shown in Table I correspond to this case. The study thus suggests that neglect of the higher order terms in a perturbation method may not always yield the correct results, as was also reported earlier in a related work [7].

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